Latent Factor Models
for Web Recommender Systems

Bee-Chung Chen
Deepak Agarwal, Pradheep Elango, Raghu Ramakrishnan
Yahoo! Research & Yahoo! Labs
Outline

• Overview of recommender problems at Yahoo!
• Basics of matrix factorization
• Matrix factorization + feature-based regression
• Matrix factorization + topic modeling
• Matrix factorization + fast online learning
• Research problems beyond factor models
  – Explore/exploit (bandit problems)
  – Offline evaluation
  – Multi-objective optimization
  – Whole-page optimization
Web Recommender Systems

Recommend **items** to **users** to maximize some **objective(s)**
Recommend search queries

Recommend packages:
- Image
- Title, summary
- Links to other pages

Pick 4 out of a pool of $K$
$K = 20 \sim 50$
to maximize clicks

Routes traffic other pages

Recommend applications

Recommend news article
Web Recommender Systems

• Goal
  – Recommend **items** to **users** to maximize some **objective(s)**

• A new scientific discipline that involves
  – Machine Learning & Statistics (for learning user-item affinity)
    • Offline Learning
    • Online Learning
    • Collaborative Filtering
    • Explore/Exploit (bandit problems)
  – Multi-Objective Optimization
    • Click-rates (CTR), time-spent, revenue
  – User Understanding
    • User profile construction
  – Content Understanding
    • Topics, “aboutness”, entities, follow-up of something, breaking news,…
Recommend packages:
- Image
- Title, summary
- Links to other pages

Pick 4 out of a pool of $K$ where $K = 20 \sim 50$ to maximize clicks

Routes traffic to other pages
CTR Curves for Two Days on Yahoo! Front Page

Each curve is the CTR of an item in the Today Module on www.yahoo.com over time.

Traffic obtained from a controlled randomized experiment (no confounding).

Things to note:
(a) Short lifetimes, (b) temporal effects, (c) often breaking news stories.
Problem Definition

Algorithm selects item $j$ with item features $x_j$ (keywords, content categories, ...)

User $i$ visits with user features $x_i$ (demographics, browse history, geo-location, search history, ...)

$(i, j) : \text{response } y_{ij}$ (click/no-click)

Which item should we select?
- The one with highest predicted CTR **Exploit**
- The one most useful for improving the CTR prediction model **Explore**
## Our Strategy

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<tr>
<th></th>
<th>Most Popular Recommendation</th>
<th>Personalized Recommendation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Offline Learning</strong></td>
<td></td>
<td>Collaborative filtering</td>
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<tr>
<td></td>
<td></td>
<td>profile construction</td>
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<td>[KDD’09, WSDM’10]</td>
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<td><strong>Online Learning</strong></td>
<td>Time-series models</td>
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<td></td>
<td>[WWW’09]</td>
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<td><strong>Intelligent Initialization</strong></td>
<td>Prior estimation</td>
<td>Prior estimation,</td>
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<td>dimension reduction</td>
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<td><strong>Explore/Exploit</strong></td>
<td>Multi-armed bandits</td>
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<td></td>
<td>[ICDM’09]</td>
<td>[Li, WWW’10]</td>
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</tbody>
</table>
Model Choices

• Feature-based (or content-based) approach
  – Use features to predict response
    • User features: Age, gender, geo-location, visit pattern, …
    • Item features: Category, keywords, topics, entities, …
    • Linear regression, Bayes Net, SVM, tree/forest methods, mixture models, …
  – Bottleneck: Need predictive features
    • Difficult to capture signals at granular levels: Cannot distinguish between users/items having same feature vectors

• Collaborative filtering (CF)
  – Make recommendation based on past user-item interaction
    • User-user, item-item, matrix factorization, …
    • See [Adomavicius & Tuzhilin, TKDE, 2005], [Konstan, SIGMOD’08 Tutorial]
  – Good performance for users and items with enough data
  – Does not naturally handle new users and new items (cold-start)
Factorization Methods

- **Matrix factorization**
  - Model each user/item as a vector of **factors** (learned from data)

\[
y_{ij} \sim \sum_k u_{ik} v_{jk} = u_i' v_j \quad \Leftrightarrow \quad Y_{M \times N} \sim U_{M \times K} V_{K \times N}
\]

\[K \ll M, N\]
\[M = \text{number of users}\]
\[N = \text{number of items}\]
Factorization Methods

• Matrix factorization
  – Model each user/item as a vector of factors (learned from data)
    \[
    y_{ij} \sim \sum_k u_{ik} v_{jk} = u'_i v_j
    \]
    where:
    \[
    \begin{align*}
    u_i & \text{ factor vector of user } i \\
    v_j & \text{ factor vector of item } j \\
    y_{ij} & \text{ rating that user } i \text{ gives item } j
    \end{align*}
    \]
    \[
    K < M, \quad N
    \]
    \[
    M = \text{ number of users} \\
    N = \text{ number of items}
    \]
  – Better performance than similarity-based methods [Koren, 2009]
  – No factor for new items/users, and expensive to rebuild the model!!

• How to prevent overfitting
• How to handle cold-start
  – Use features (given) to predict the factor values
How to Prevent Overfitting

• Loss minimization

\[ \ell(u, v) = \]

\[ \frac{1}{2\sigma^2} \sum_{(i,j)} (y_{ij} - u_i'v_j)^2 \]

\[ + \frac{1}{2\sigma_u^2} \sum_i \|u_i\|^2 \]

\[ + \frac{1}{2\sigma_v^2} \sum_j \|v_j\|^2 \]

Given \( \sigma^2, \sigma_u^2, \sigma_v^2 \), find

\[ \arg\min_{u,v} \ell(u, v) \quad \text{equivalent} \quad \arg\max_{u,v} \Pr[u, v | y] \]

How to set \( \sigma^2, \sigma_u^2, \sigma_v^2 \) ?

• Probabilistic model

\[ y_{ij} \sim N(u_i'v_j, \sigma^2) \]

\[ u_i \sim N(0, \sigma_u^2 I) \]

\[ v_j \sim N(0, \sigma_v^2 I) \]
Probabilistic Matrix Factorization

- Probabilistic model

\[ y_{ij} \sim N(u_i'v_j, \sigma^2) \]
\[ u_i \sim N(0, \sigma_i^2 I) \]
\[ v_j \sim N(0, \sigma_j^2 I) \]

Let \( \Theta = (\sigma^2, \sigma_u^2, \sigma_v^2) \)

\[
\log \Pr(y, u, v | \Theta) = \text{constant}
- \frac{1}{2\sigma^2} \sum_{(i,j)} (y_{ij} - u_i'v_j)^2 - R \log \sigma^2
- \frac{1}{2\sigma_u^2} \sum_i \|u_i\|^2 - Mr \log \sigma_u^2
- \frac{1}{2\sigma_v^2} \sum_j \|v_j\|^2 - Nr \log \sigma_v^2
\]

How to determine \( \Theta \)?
- Maximum likelihood estimate

\[
\arg \max_\Theta \Pr(y | \Theta) = \arg \max_\Theta \int \Pr(y, u, v | \Theta) \, du \, dv
\]
- Use the EM algorithm
Model Fitting: EM Algorithm

• Find

$$\hat{\Theta} = \arg \max_{\Theta} \Pr(y \mid \Theta) = \arg \max_{\Theta} \int \Pr(y, u, v \mid \Theta) \, du \, dv$$

• Iterate between E-step and M-step until convergence
  – Let $$\hat{\Theta}^{(n)}$$ be the current estimate
  – E-step: Compute

$$f(\Theta) = E_{(u, v \mid y, \hat{\Theta}^{(n)})} [\log \Pr(y, u, v \mid \Theta)]$$

$$- \frac{1}{2\sigma^2} \sum_{(i, j)} E[(y_{ij} - u'_i v_j)^2] - \frac{1}{2\sigma_u^2} \sum_i E \| u_i \|^2 - \frac{1}{2\sigma_v^2} \sum_j E \| v_j \|^2$$

$$- R \log \sigma^2 - Mr \log \sigma_u^2 - Nr \log \sigma_v^2$$

• The expectation is not in closed form
• We draw Gibbs samples and compute the Monte Carlo mean

• M-step: Find

$$\hat{\Theta}^{(n+1)} = \arg \max_{\Theta} f(\Theta)$$
Example: timeSVD++

- Example of matrix factorization in practice
- Part of the winning method of Netflix contest [Koren 2009]

\[
y_{ij,t} \sim \mu + b_i(t) + b_j(t) + u_i(t)'v_j
\]

- item popularity
- user bias
- user factors (preference)

\[
b_i(t) = b_i + \alpha_i \text{ dev}_i(t) + b_{it}
\]

- distance to the middle rating time of \(i\)

\[
b_j(t) = b_j + b_{j,\text{bin}(t)}
\]

- time bin

\[
u_i(t)_k = u_{ik} + \alpha_{ik} \text{ dev}_u(t) + u_{ikt}
\]

Model parameters: \(\mu, b_i, \alpha_i, b_{it}, b_j, b_{jd}, u_{ik}, \alpha_{ik}, u_{ikt}\)

for all user \(i\), item \(j\), factor \(k\), time \(t\), time bin \(d\)
How to Handle Cold Start?

- For new items and new users, their factor values are all 0
- Simple idea
  - Predict their factor values based on features
    - For new user $i$, predict $u_i$ based on $x_i$ (user feature vector)

\[ u_i \sim G x_i \]

- An item may be represented by a bag of words (later)
RLFM: Regression-based Latent Factor Model

- Incorporate features into matrix factorization
  - $x_i$: feature vector of user $i$
  - $x_j$: feature vector of item $j$

- Probabilistic model
  \[
  y_{ij} \sim N(u_i'v_j, \sigma^2) \\
  u_i \sim N(Gx_i, \sigma_u^2 I) \\
  v_j \sim N(Dx_j, \sigma_v^2 I)
  \]

Let $\Theta = (G, D, \sigma^2, \sigma_u^2, \sigma_v^2)$

\[
\log \Pr(y, u, v | \Theta) = \text{constant} \\
- \frac{1}{2\sigma^2} \sum_{(i,j)} (y_{ij} - u_i'v_j)^2 - R \log \sigma^2 \\
- \frac{1}{2\sigma_u^2} \sum_i \| u_i - Gx_i \|^2 - Mr \log \sigma_u^2 \\
- \frac{1}{2\sigma_v^2} \sum_j \| v_j - Dx_j \|^2 - Nr \log \sigma_v^2
\]

Find

\[
\hat{\Theta} = \arg \max_{\Theta} \Pr(y | \Theta) = \arg \max_{\Theta} \int \Pr(y, u, v | \Theta) \, du \, dv
\]
Comparison

- Zero-mean factorization
  \[ y_{ij} \sim N(u_i'v_j, \sigma^2) \]
  \[ u_i \sim N(0, \sigma_u^2 I) \]
  \[ v_j \sim N(0, \sigma_v^2 I) \]

- Factorization with features (RLFM)
  \[ y_{ij} \sim N(u_i'v_j, \sigma^2) \quad y_{ij} \sim N(x_i'G'Dx_j + \delta_i'Dx_j + x_i'G'\eta_j + \delta_i'\eta_j, \sigma^2) \]
  \[ u_i \sim N(Gx_i, \sigma_u^2 I) \quad u_i = Gx_i + \delta_i, \quad \delta_i \sim N(0, \sigma_u^2 I) \]
  \[ v_j \sim N(Dx_j, \sigma_v^2 I) \quad v_j = Dx_j + \eta_j, \quad \eta_j \sim N(0, \sigma_v^2 I) \]

- Feature-only model
  \[ y_{ij} \sim N(x_i'G'Dx_j, \sigma^2) \]
Illustration

Factorization with features

Factorization without feature
Non-linear RLFM

rating that user $i$ gives item $j$

$$y_{ij} \sim b(x'_{ij}) + \alpha_i + \beta_j + u_i \cdot v_j$$

$x_i$ = feature vector of user $i$

$x_j$ = feature vector of item $j$

$x_{ij}$ = feature vector of $(i, j)$

- **Bias of user $i$:** $\alpha_i = g(x_i) + \varepsilon_i^\alpha$, $\varepsilon_i^\alpha \sim N(0, \sigma_\alpha^2)$
- **Popularity of item $j$:** $\beta_j = d(x_j) + \varepsilon_j^\beta$, $\varepsilon_j^\beta \sim N(0, \sigma_\beta^2)$
- **Factors of user $i$:** $u_i = G(x_i) + \varepsilon_i^u$, $\varepsilon_i^u \sim N(0, \sigma_u^2 I)$
- **Factors of item $j$:** $v_i = D(x_j) + \varepsilon_i^v$, $\varepsilon_i^v \sim N(0, \sigma_v^2 I)$

$b, g, d, G, D$ are regression functions

*Any regression model can be used here!!*
fLDA: Factorization through LDA Topic Model

• An item is represented by a bag of words
• Model the rating $y_{ij}$ that user $i$ gives to item $j$ as the user’s affinity to the topics that the item has

\[
y_{ij} = \ldots + \sum_k s_{ik} \bar{z}_{jk}
\]

User $i$’s affinity to topic $k$

\[\Pr(\text{item } j \text{ has topic } k) \text{ estimated by averaging the LDA topic of each word in item } j\]

The topic distribution $z_{jk}$ of a new item $i$ is predicted based on the bag of words in the item

– Unlike regular unsupervised LDA topic modeling, here the LDA topics are learnt in a supervised manner based on past rating data
– These supervised topics are likely to be more useful for the prediction purpose
Supervised Topic Assignment

The topic of the $n$th word in item $j$

$$\Pr(z_{jn} = k \mid \text{Rest})$$

$$\propto \frac{Z_{kl}^{-jn} + \eta}{Z_k^{-jn} + W\eta} (Z_{jk}^{-jn} + \lambda) \cdot \prod_{i \text{ rated } j} f(y_{ij} \mid z_{jn} = k)$$

Same as unsupervised LDA

Probability of observing $y_{ij}$ given the model

Likelihood of observed ratings by users who rated item $j$ when $z_{jn}$ is set to topic $k$
Experimental Results (MovieLens)

- **Task:** Predict the rating that a user would give a movie

- **Training/test split:**
  - Sort observations by time
  - First 75% → Training data
  - Last 25% → Test data

- **User cold-start scenario**
  - 56% test data with new users
  - 2% new items in test data

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<tr>
<th>Model</th>
<th>Test RMSE</th>
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</thead>
<tbody>
<tr>
<td>RLFM</td>
<td>0.9363</td>
</tr>
<tr>
<td>fLDA</td>
<td>0.9381</td>
</tr>
<tr>
<td>Factor-Only</td>
<td>0.9422</td>
</tr>
<tr>
<td>FilterBot</td>
<td>0.9517</td>
</tr>
<tr>
<td>unsup-LDA</td>
<td>0.9520</td>
</tr>
<tr>
<td>MostPopular</td>
<td>0.9726</td>
</tr>
<tr>
<td>Feature-Only</td>
<td>1.0906</td>
</tr>
<tr>
<td>Constant</td>
<td>1.1190</td>
</tr>
</tbody>
</table>
Summary

• Factorization methods usually have better performance than pure feature-based methods
  – Netflix
  – Our experience

• Metadata (feature vector or bag of words) can be easily incorporated into matrix factorization

• Next step
  – Matrix factorization with social networks
    • Friendship: Address book
    • Communication: Instant messages, emails
  – Multi-application factorization
    • E.g., joint factorization of the (user, news article) matrix and the (user, query) matrix
Fast Online Learning for Time-sensitive Recommendation

• Examples of time-sensitive items
  – News stories, trending queries, tweets, updates, events …

• Real-time data pipeline that continuously collects new ratings (clicks) on new items

• Modeling requirements:
  – **Fast learning**: Learn good models for new items using little data
    • Good initial guess (without ratings on new items)
    • Fast convergence
  – **Fast computation**: Build good models using little time
    • Efficient
    • Scalable
    • Parallelizable
FOBFM: Fast Online Bilinear Factor Model

Per-item online model\[ y_{ij} \sim u'_i \beta_j, \quad \beta_j \sim N(\mu_j, \Sigma) \]

- Feature-based model initialization\[ \beta_j \sim N(Ax_j, \Sigma) \quad \Leftrightarrow \quad y_{ij} \sim u'_i Ax_j + u'_i v_j \]
  - predicted by features\[ v_j \sim N(0, \Sigma) \]
- Dimensionality reduction for fast model convergence\[ v_j = B \theta_j \quad B \text{ is a } n \times k \text{ linear projection matrix } (k << n) \]
  - project: high dim($v_j$) $\rightarrow$ low dim($\theta_j$)\[ \theta_j \sim N(0, \sigma^2 \mathbb{I}) \quad \text{low-rank approx of } \text{Var}[\beta_j]: \quad \beta_j \sim N(Ax_j, \sigma^2 \theta BB') \]

\[
\begin{bmatrix}
v_j \\
B \\
\theta_j
\end{bmatrix} =
\]
Offline training: Determine $A$, $B$, $\sigma^2_\theta$ (once per day)

Subscript:
- user $i$
- item $j$

Data:
- $y_{ij}$ = rating that user $i$ gives item $j$
- $u_i$ = offline factor vector of user $i$
- $x_j$ = feature vector of item $j$
FOBFM: Fast Online Bilinear Factor Model

**Per-item online model**

\[ y_{ij} \sim u_i' \beta_j, \quad \beta_j \sim N(\mu_j, \Sigma) \]

- Feature-based model initialization

\[ \beta_j \sim N(Ax_j, \Sigma) \quad \iff \quad y_{ij} \sim u_i'Ax_j + u_i'v_j \]

predicted by features

\[ v_j \sim N(0, \Sigma) \]

- Dimensionality reduction for fast model convergence

\[ v_j = B\theta_j \]

\[ \theta_j \sim N(0, \sigma^2 \theta I) \]

\(B\) is a \(n \times k\) linear projection matrix \((k << n)\)

project: high \(\text{dim}(v_j) \rightarrow \text{low dim}(\theta_j)\)

low-rank approx of \(\text{Var}[\beta_j]: \beta_j \sim N(Ax_j, \sigma^2 \theta BB')\)

- Fast, parallel online learning

\[ y_{ij} \sim u_i'Ax_j + (u_i'B)\theta_j, \quad \text{where } \theta_j \text{ is updated in an online manner} \]

offset

new feature vector (low dimensional)

- Online selection of dimensionality \((k = \text{dim}(\theta_j))\)

  - Maintain an ensemble of models, one for each candidate dimensionality

Subscript:
- user \(i\)
- item \(j\)

Data:
- \(y_{ij}\) = rating that user \(i\) gives item \(j\)
- \(u_i\) = offline factor vector of user \(i\)
- \(x_j\) = feature vector of item \(j\)
Experimental Results: My Yahoo! Dataset (1)

- My Yahoo! is a personalized news reading site
  - Users manually select news/RSS feeds
- ~12M “ratings” from ~3M users to ~13K articles
  - Click = positive
  - View without click = negative
Experimental Results: My Yahoo! Dataset (2)

- Item-based data split: Every item is new in the test data
  - First 8K articles are in the training data (offline training)
  - Remaining articles are in the test data (online prediction & learning)

- Our supervised dimensionality reduction (reduced rank regression) significantly outperforms other methods

Methods:
- **No-init**: Standard online regression with ~1000 parameters for each item
- **Offline**: Feature-based model without online update
- **PCR, PCR+**: Two principal component methods to estimate $B$
- **FOBFM**: Our fast online method
Experimental Results: My Yahoo! Dataset (3)

- Small number of factors (low dimensionality) is better when the amount of data for online learning is small.
- Large number of factors is better when the data for learning becomes large.
- The online selection method usually selects the best dimensionality.

# factors = Number of parameters per item updated online

![Graph showing lift in test log likelihood versus number of observations per item for different numbers of factors.](image)
Experimental Results: MovieLens Dataset

- Training-test data split
  - Time-split: First 75% ratings in training; rest in test
  - Movie-split: 75% randomly selected movies in training; rest in test

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<th>RMSE Time-split</th>
<th>RMSE Movie-split</th>
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<tr>
<td>FOBFM</td>
<td>0.8429</td>
<td>0.8549</td>
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<tr>
<td>RLFM</td>
<td>0.9363</td>
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<td>Online-UU</td>
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<td>0.9453</td>
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<tr>
<td>Constant</td>
<td>1.1190</td>
<td>1.1162</td>
</tr>
</tbody>
</table>

FOBFM: Our fast online method
RLFM: [Agarwal 2009]
Online-UU: Online version of user-user collaborative filtering
Online-PLSI: [Das 2007]
Experimental Results: Yahoo! Front Page Dataset

• Training-test data split
  – Time-split: First 75% ratings in training; rest in test

  ~2M “ratings” from ~30K frequent users to ~4K articles
  • Click = positive
  • View without click = negative

  – Our fast learning method outperforms others
Summary

• Recommending time-sensitive items is challenging
  – Most collaborative filtering methods do not work well in cold start
  – Rebuilding models can incur too much latency when the numbers of items and users are large

• Our approach:
  – Periodically rebuild the offline model that
    • uses feature-based regression to predict the initial point for online learning, and
    • reduces the dimensionality of online learning
  – Rapidly update online models once new data is received
    • Fast learning: Low dimensional and easily parallelizable
    • Online selection for the best dimensionality
Important Problems Beyond Factor Models

• How to explore/exploit with small traffic, a large item pool, at a fine granularity
• Offline evaluation
• Multi-objective optimization under uncertainty
• Whole page optimization
Explore/Exploit

Determine \((x_1, x_2, \ldots, x_K)\) based on clicks and views observed before \(t\) in order to maximize the expected total number of clicks in the future.

- Large number of items
- Small traffic
- Deep personalization

ICDM’09 (best paper)
- Small number of items
- No deep personalization

Challenges
Offline Evaluation

- Ultimate evaluation: Online bucket test
- Unbiased offline evaluation based on random-bucket data
  - [Lihong Li, WWW’10, WSDM’11]
  - Random bucket: A small user population to which we show each item with equal probability
  - Assumptions:
    - Single recommendation per visit (instead of top-$K$)
    - All the users respond to the recommended item in an iid manner
  - Replay-match methodology
- Challenges
  - How to handle non-random data
  - How to extend to top-$K$ recommendation
  - How to capture users’ “non-iid” behavior in a session
Multi-Objective Optimization

- Maximize time-spent (or revenue) s.t. click drop < 5%

Challenges:
- Deep personalization
- Optimization in the presence of uncertainty
Whole Page Optimization

Challenge:
How to jointly optimize all these modules
- Diversity
- Consistency
- Relatedness
Thank You!

Contact me for job/internship opportunities in Yahoo! Labs
beechun@yahoo-inc.com